Arbitrageurs' profits, LVR, and sandwich attacks: batch trading as an AMM design response

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Blockchain@X - OMI Workshop

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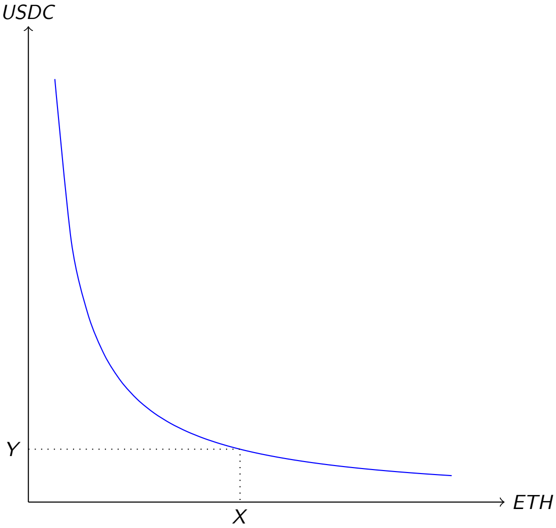
- Most common invariant function: the product $\Rightarrow$ $p(x) = y/x = Y/(X - x)$

Constant Product Automated Market Maker



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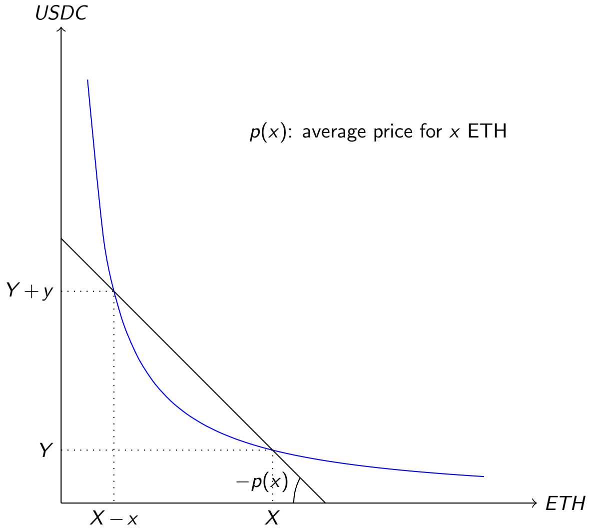

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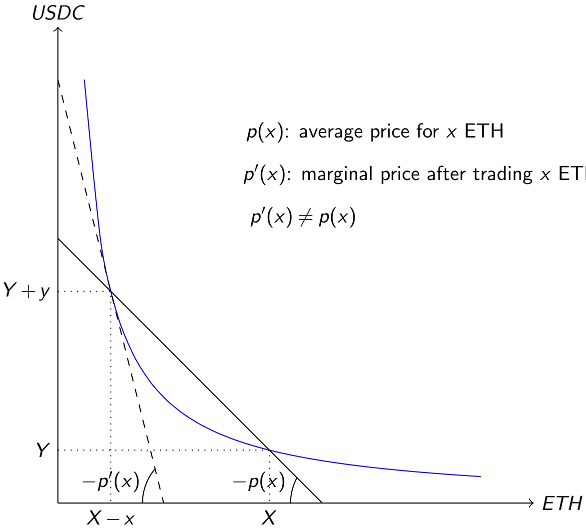

$p(x)$: average price for x ETH

$p'(x)$: marginal price after trading x ETH

$p'(x) \neq p(x)$

Problem with Constant function AMM: (1) Arbitrage Profits (aka Loss-vs-rebalancing, aka LVR)

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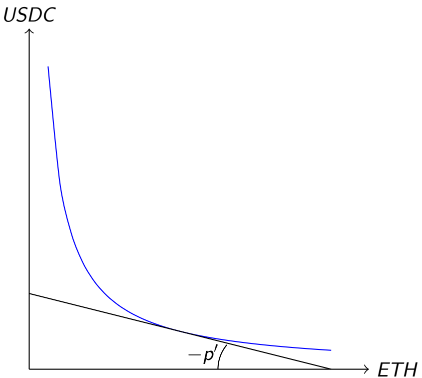

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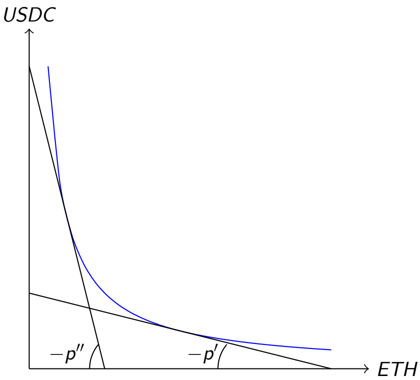

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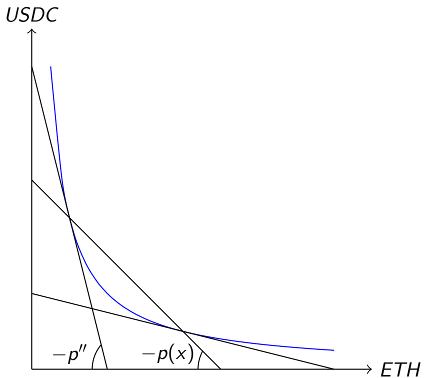

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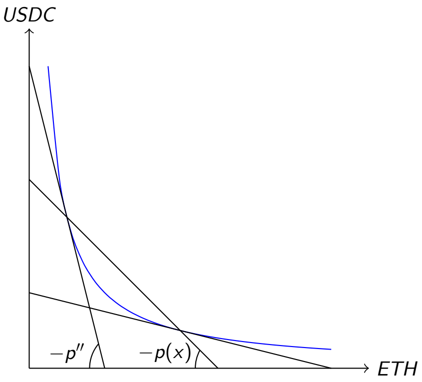
- Defensive mechanism: fees



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- back run the victim with the opposite trade (sell ETH)
- $\Rightarrow$ the attacker buys cheap and sells expensive; the victim buys expensive

Our paper: batching trades (and designing the AMM around this) solves both problems

- All trades are collected off-chain and batched:
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The Function Maximizing AMM (FM-AMM) with product function

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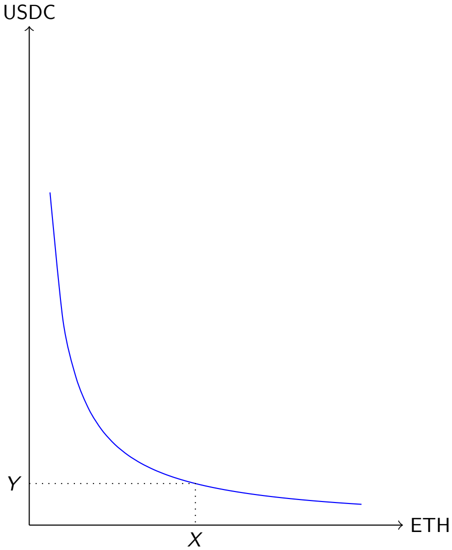
Hence, to purchase x ETH on the FM-AMM, the price needs to be:

$$p^{FM-AMM}(x) = \frac{Y}{X - 2x}.$$

The FM-AMM "moves up the curve"



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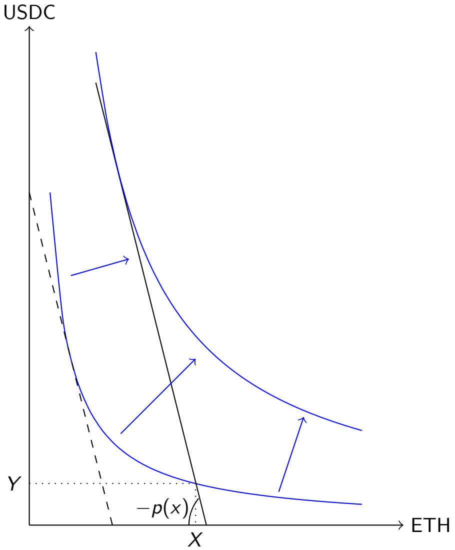

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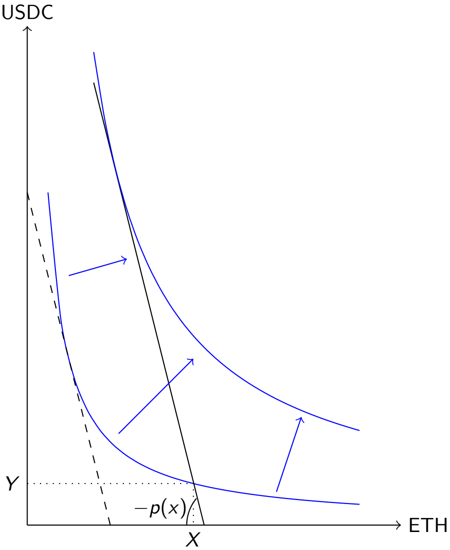

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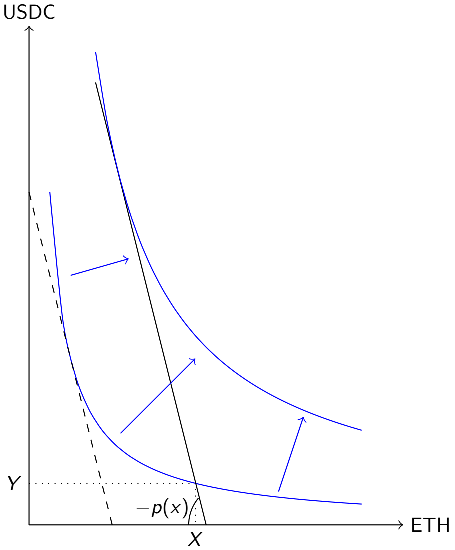

- FM-AMM is **clearing-price consistent**: the price at which it trades equals the marginal price after the trade
- FM-AMM violates path independence: it can be exploited by splitting trades → **batching is necessary**

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- FM-AMM charges no fee for inclusion in a batch, and a fee τ (in the input token) for settling an order on the FM-AMM

Theoretical model: FM-AMM in equilibrium

Proposition:

Suppose that, at the end of an even period, the reserves of the FM-AMM are X and Y. In the equilibrium of the subsequent odd period, after p^* is realized, if noise traders collectively submit trade A to the batch, then arbitrageurs will collectively submit trade $y(p^*)$ such that

$$\tilde{p}(A + y(p^*), \tau) = p^*$$

where $\tilde{p}(A + y(p^*), \tau)$ is the FM-AMM effective price (i.e., price after fees).

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- p^* determines the rebalancing trade, which determines the fees earned

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AMM function		
Value of reserves		

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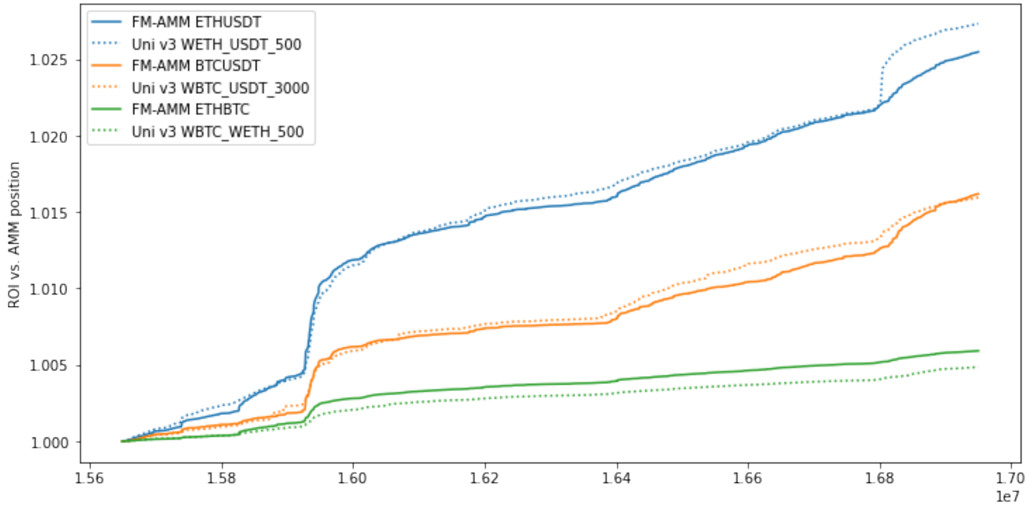
- Compare the two.

FM-AMM vs Uniswap v3 pool



FM-AMM vs Uniswap v3 pool

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- maximum difference in value between the two liquidity positions (expressed in percentage of the initial liquidity position) is 0.30% (for the ETH-USDT pair), 0.14% (for the BTC-USDT pair) and 0.12% (for the ETH-BTC pair).

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- **The differences in returns are small**.

Conclusions

- Batching allows for a novel AMM design that eliminates arbitrageurs' profits (LVR) and sandwich attacks.
- (for the period and the token pair we consider) for liquidity providers, an FM-AMM that does not earn fees from noise traders performs as well as Uniswap v3
    - An FM-AMM that also earns fees from noise traders should perform better

Literature

- Batch auction: Budish, Cramton, and Shim (2015)
- Sandwich attacks: Park (2022), Torres at al. (2021), Qin et al. (2022).
- Arb profits and LVR: Aoyagi (2020), Capponi and Jia (2021), and Milionis et al. (2022), Milionis et al. (2023)
- surplus maximizing AMM / axiomatization of AMM: Goyal et al. (2022), Schlegel and Mamageishvili (2022)
- several blog posts

Thank you!