Fundamental Pricing of Utility Tokens

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Utility Tokens

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Merriam-Webster SINCE 1828

utility token

DICTIONARY THESAURUS

utility token noun

Save Word

Definition of *utility token*

: a digital token of cryptocurrency that is issued in order to fund development of the cryptocurrency and that can be later used to purchase a good or service offered by the issuer of the cryptocurrency

// sold *utility tokens* as a method of fundraising for the start-up

First Known Use of *utility token*

2016, in the meaning defined above

Utility Tokens

- Tokens are like
  - equity because no risk of bankruptcy,
  - debt because no dilution of control

- Cannot be priced using off-the-shelf techniques


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How to Price Utility Tokens

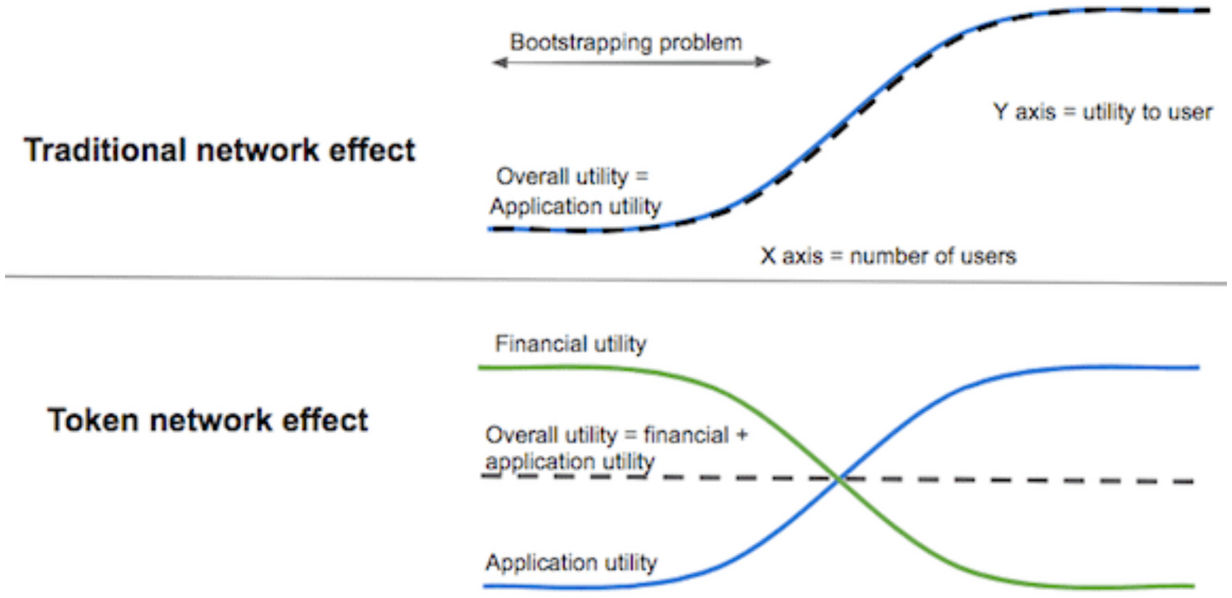
- Sidestep corporate finance issues to focus on pricing
- Outline valuation framework based on fundamentals
- Dynamic model that relates price of tokens to observable statistics such as:
    - Share of tokens held by active users,
    - Velocity of circulation

Contribution

- Crypto-investors use ad-hoc pricing formula based on the quantity theory of money
- We rely instead on the **token-in-advance constraint**
- Endogenize velocity of circulation of tokens
- Step towards microfounded pricing models for the benchmarking and rating of ICOs

Insights

- Help rationalize frameworks used by practitioners

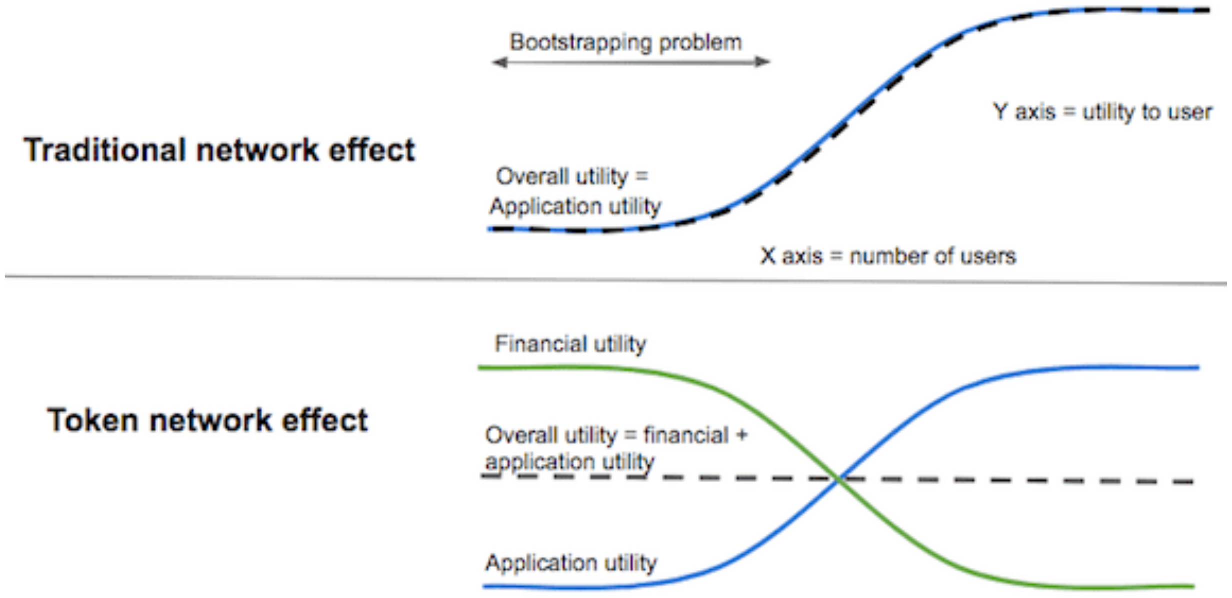


Source: Chris Dixon (a16z), The Web3 Playbook

Insights

- Help rationalize frameworks used by practitioners:

1. Tokens foster adoption because they yield financial returns
2. Speculative regime followed by user adoption
3. Velocity of circulation correlated with valuation



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Insights

- Help explain the "*irrational exuberance*" of crypto markets:

    1. Excess price volatility and high returns during early speculative phase

    2. Significant valuation in spite of marginal user adoption

Related Literature

- **Valuation of cryptocurrencies**: Gans and Halaburda (2015), Garratt and Wallace (2018), Buraschi and Pagnotta (2018), Sockin and Xiong (2018), Chiu and Koeppl (2019), Schilling and Uhlig (2019), Biais et al. (2020), Huberman, Leshno and Moallemi (2020)

- **ICO design and incentives**: Bakos and Halaburda (2018), Canidio (2018), Catalini and Gans (2018), Chod and Lyandres (2018), Malinova and Park (2018), Garratt and Van Oordt (2019), Goldstein et al. (2019), Howell, Niessner and Yermack (2019), Davydiuk et al. (2020), Gan et al. (2020), Li and Mann (2020)

- **User adoption and fundamental pricing**: Athey et al. (2016), Cong, Li and Wang (2018, 2020)

Set Up

- Two markets:

  (i) **Good market** where tokens are exchanged against the platform's output

  (ii) **Financial market** where tokens are bought using fiat money

Decentralization: Output is produced by contributors (e.g. miners) to the platform

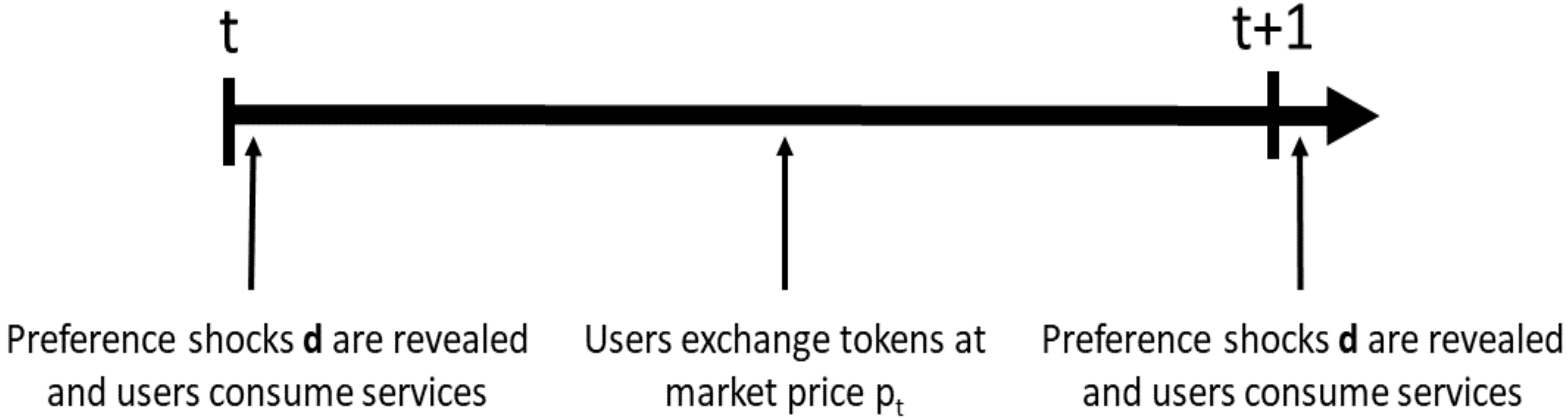
Supply from contributors is (linearily) increasing in fiat value of payments

Users Preferences

- A constant share $\lambda \in (0,1)$ of users need the service in each period

$$\mathcal{U}\left(c, d^i\right) = u(c) * d^i, \text{ where } d^i = \begin{cases} 0 \text{ with probability } 1-\lambda \\ 1 \text{ with probability } \lambda \end{cases}.$$

- Timing:



Token Holdings

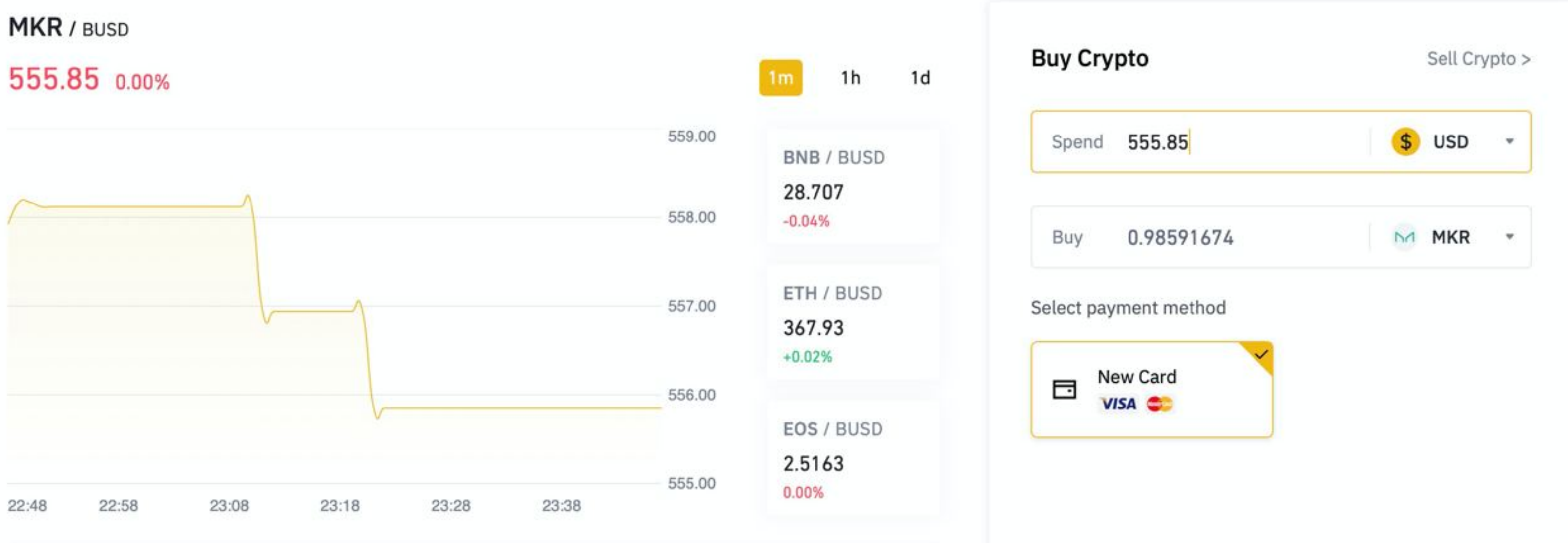
- **m** = Token holdings
- **λ** = Probability at which service is needed
- **r** = Interest rate
- **u(c)** = Utility as a function of consumption c
- **P_t** = Price of token in fiat at date t

- Utility flow

$$v(p_t, p_{t+1}) \equiv \max_{m \geq 0} \left\{ \lambda \max_{c \in [0, m p_{t+1}]} \left\{ u(c) + p_{t+1} \left(m - \frac{c}{p_{t+1}} \right) \right\} + (1 - \lambda) m p_{t+1} - (1 + r) m p_t \right\}$$

Token-in-Advance Constraint

- The token-in-advance constraint is not a theoretical figment but an actual description of how the technology operates
- Decentralized applications and smart-contracts often require an immediate reaction
- Significant transaction fees for buying tokens on the fly



Source: Binance

Law-of-Motion of Price

- Minimize token holdings (*c=mp)*

$$v\left(p_t, p_{t+1}\right) = \max_{m_t \geq 0} \left\{\lambda u\left(m_t p_{t+1}\right) + (1-\lambda) m_t p_{t+1} - (1+r) m_t p_t\right\}$$

hence price obeys

$$r p_t = \underbrace{\lambda p_{t+1}\left[u'\left(m_t^* p_{t+1}\right) - 1\right]}_{\text{Convenience Yield}} + \underbrace{p_{t+1} - p_t}_{\text{Capital Gain}}.$$

- Value for users is proportional to the discounted surplus of their next trade

Equilibrium Price

- Price level adjusts downwards until market clears

- Steady-state price reads

$$\hat{p} = \frac{1}{M} u'^{-1} \left(\frac{r + \lambda}{\lambda} \right).$$

➡ Neutrality of token mass M

Gradual Adoption

- Demand shifter z captures technological progress

$$u(c; z) = zu(c; 1).$$

- Shifter follows a Geometric Brownian Motion

$$dz_t = z_t(\mu dt + \sigma dB_t)$$

- Law-of-motion of price in continuous time ($\dot{p}_t = E[dp_t/dt]$)

$$\begin{aligned} v(p_t, \dot{p}_t, z_t) &= \lambda [z_t u(p_t m_t^*) - p_t m_t^*] + \dot{p}_t m_t^* - r p_t m_t^*, \\ m_t^* &= \frac{1}{p_t} u'^{-1}\left(\frac{1}{\lambda z_t}\left[r + \lambda - \frac{\dot{p}_t}{p_t}\right]\right), \end{aligned}$$

Heterogenous Users

- Users with different technological proficiency .

$$\chi^i \sim G(\chi)$$

- User i hoards some tokens if and only if

$$v(\dot{p}_t/p_t, z_t) \geq \chi_i^{-1}.$$

- User base

$$N\left(\frac{\dot{p}_t}{p_t}, z_t\right) = 1 - G\left(\frac{1}{v\left(\frac{\dot{p}_t}{p_t}, z_t\right)}\right).$$

Dynamic Model

- Appreciation rate of tokens cannot exceed r
- Hence token holdings and number of users are bounded

$$m_t^* \leq \overline{m}(p_t, z_t) \equiv u'^{-1}\left(1/z_t\right)/p_t$$

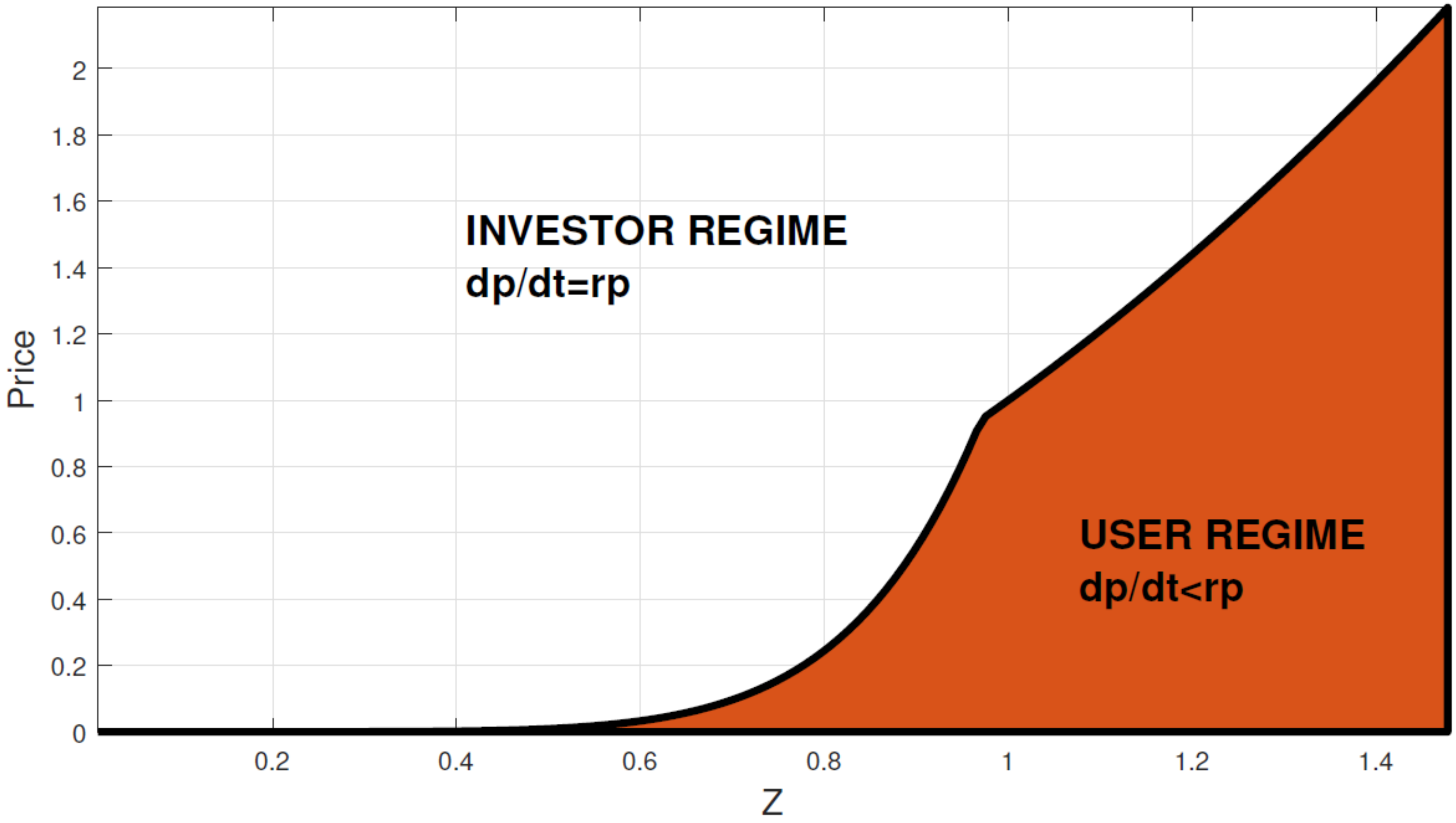
$$N_t \leq \overline{N}(z_t) \equiv N\left(r, z_t\right)$$

- There exists a *unique price*

$$\overline{p}(z) = u'^{-1}\left(\frac{1}{z}\right)\frac{\overline{N}(z)}{M}$$

such that users' demand *clears the market* iff $p(z) \leq \bar{p}(z)$

Regimes



INVESTOR REGIME
dp/dt=rp
USER REGIME
dp/dt<rp
Price
Z
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4

Law-of-Motion of Price

- Law-of-motion depends on whether marginal holder is a user or an investor:
  - **If the marginal holder is a user**

$$\dot{p}_t = -\lambda p_t \left[z_t u' \left(p_t \frac{M}{N_t} \right) - 1 \right] + r p_t, \text{ when } p_t \leq \overline{p}(z_t)$$

  - **If the marginal holder is an investor**

$$\dot{p}_t = r p_t, \text{ when } p_t > \overline{p}(z_t)$$

Fundamental Solution

Proposition 1 *A Markov equilibrium with state variable z_t is a price function $p(z_t)$ that obeys the law-of-motion*

$$rp(z_t) = \begin{cases} -\lambda p(z_t) + z_t \mu p'(z_t) + \dfrac{(z_t\sigma)^2}{2} p''(z_t) + \lambda z_t p(z_t) u'\left(p(z_t)\dfrac{M}{N_t}\right), \text{ when } p(z_t) \leq \overline{p}(z_t), \\ z_t \mu p'(z_t) + \dfrac{(z_t\sigma)^2}{2} p''(z_t), \text{ when } p(z_t) > \overline{p}(z_t), \end{cases}$$

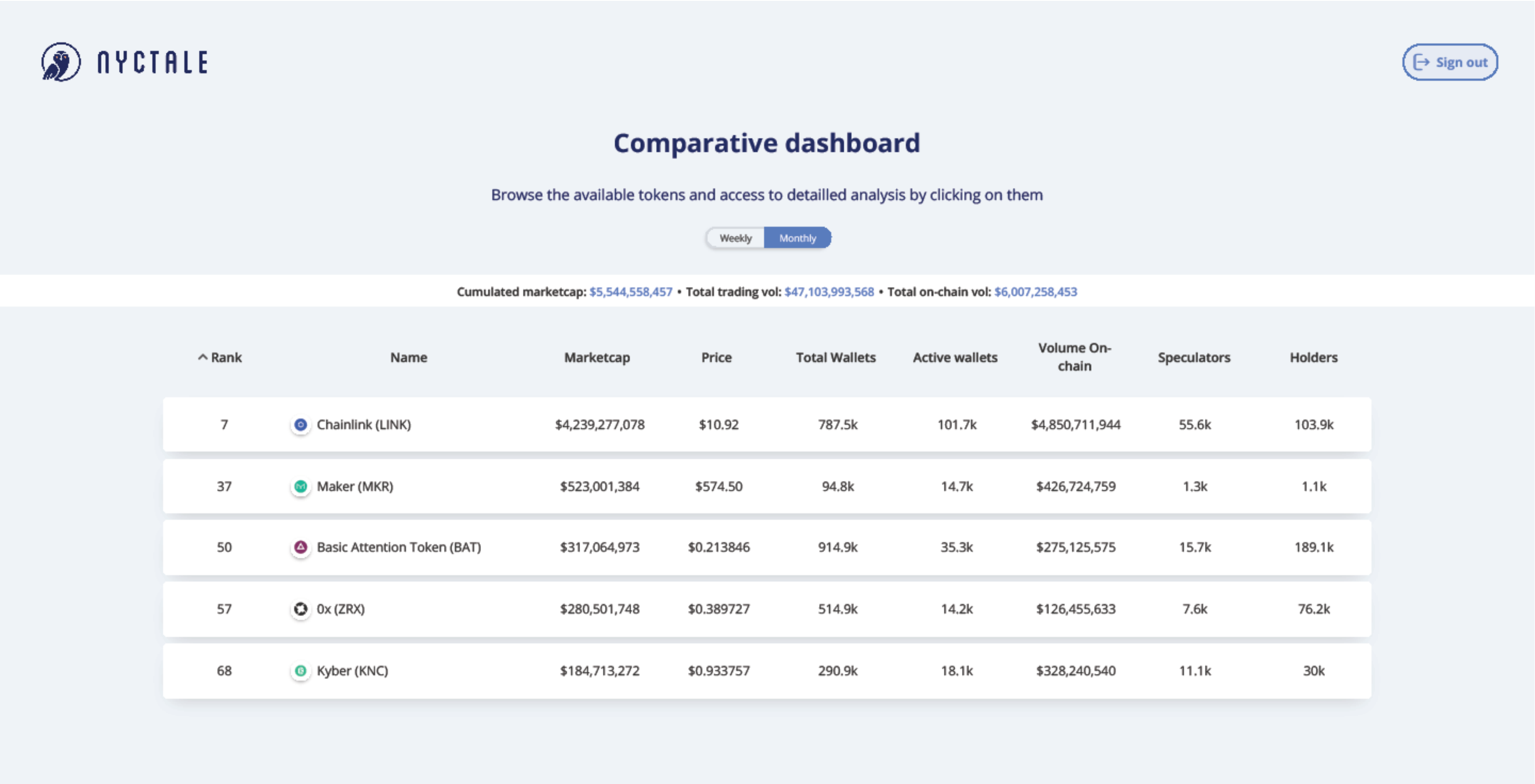
where N_t satisfies the participation constraint of users.

Provided that (i) $\tilde{\mu} \equiv \left[\mu + (1/\eta - 1)\sigma^2/2\right]/\eta < r$ and (ii) the utility function of users is CRRA; the fundamental solution also satisfies the following two boundary conditions $p(0) = 0$ and

$$\lim_{z_t \to \infty} p(z_t) = \tilde{p}(z_t) = \left[\frac{\lambda z_t}{r + \lambda - \tilde{\mu}}\right]^{\frac{1}{\eta}} \frac{1}{M}.$$

Calibration

Partnership with start-up Nyctale that collects data on wallets



NYCTALE

Sign out

Comparative dashboard

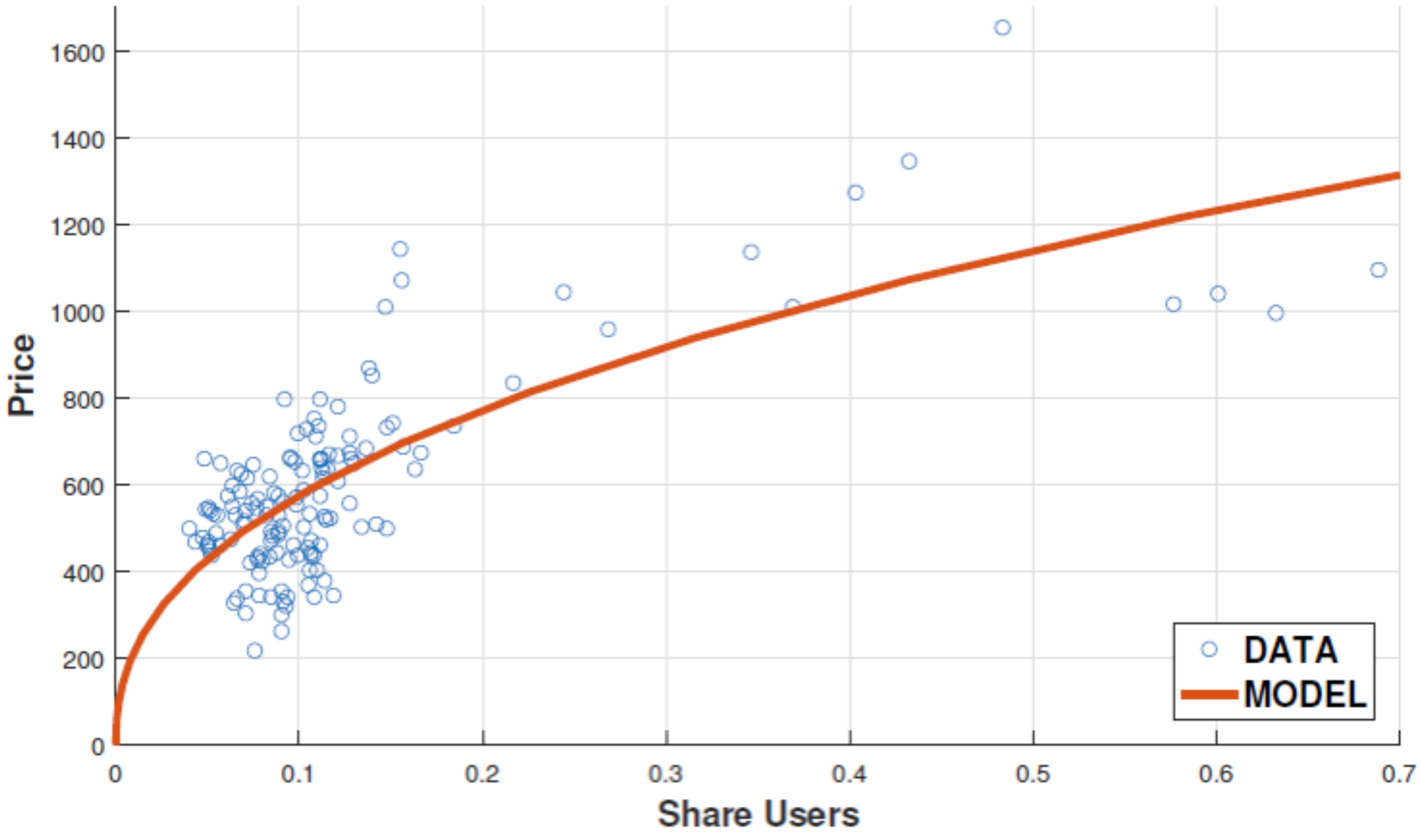
Browse the available tokens and access to detailled analysis by clicking on them

Weekly | Monthly

Cumulated marketcap: $5,544,558,457 • Total trading vol: $47,103,993,568 • Total on-chain vol: $6,007,258,453

Rank	Name	Marketcap	Price	Total Wallets	Active wallets	Volume On-chain	Speculators	Holders
7	Chainlink (LINK)	$4,239,277,078	$10.92	787.5k	101.7k	$4,850,711,944	55.6k	103.9k
37	Maker (MKR)	$523,001,384	$574.50	94.8k	14.7k	$426,724,759	1.3k	1.1k
50	Basic Attention Token (BAT)	$317,064,973	$0.213846	914.9k	35.3k	$275,125,575	15.7k	189.1k
57	0x (ZRX)	$280,501,748	$0.389727	514.9k	14.2k	$126,455,633	7.6k	76.2k
68	Kyber (KNC)	$184,713,272	$0.933757	290.9k	18.1k	$328,240,540	11.1k	30k

Calibration on Maker Token



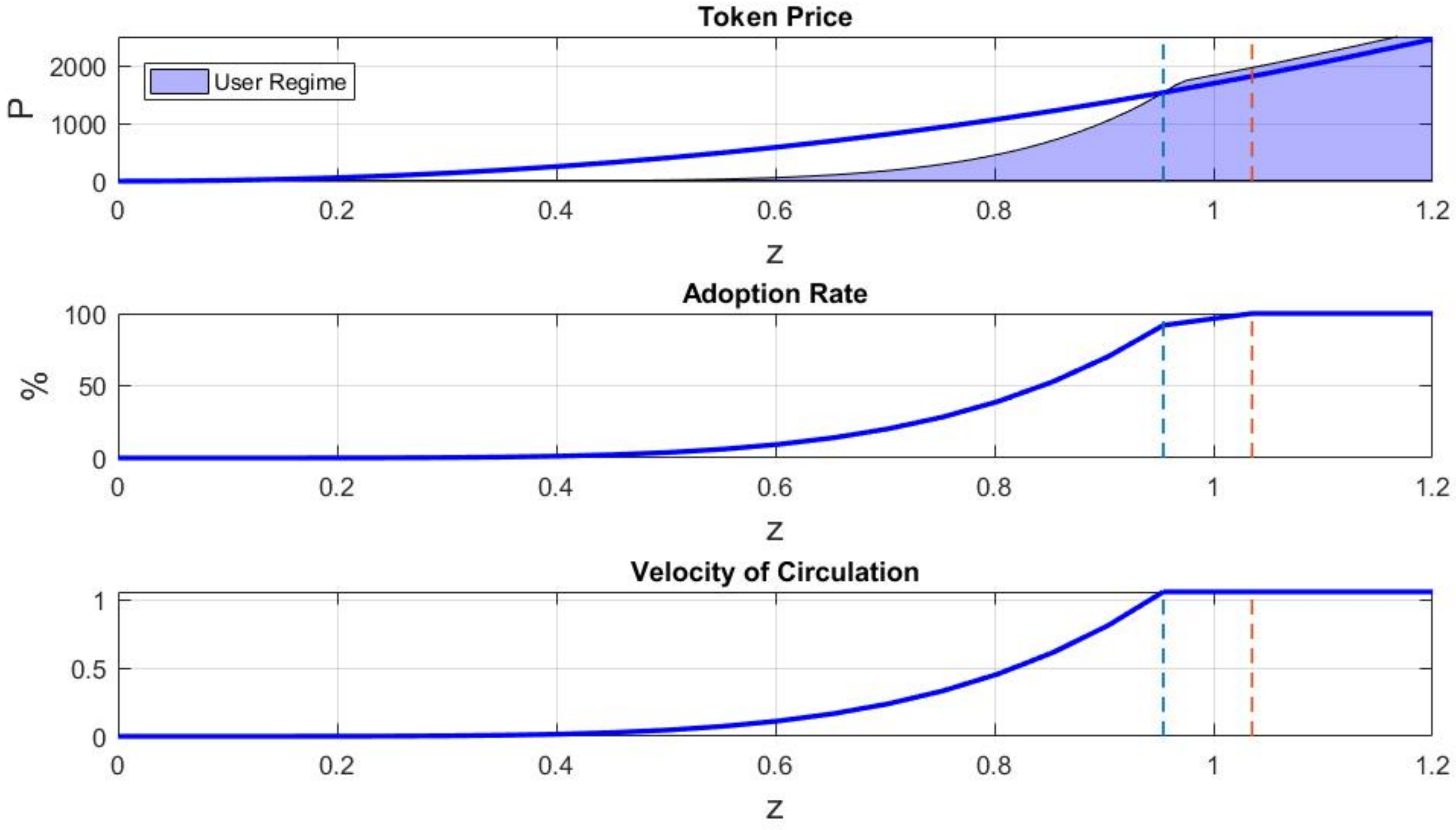
Token Price as a Function of the Share of Tokens Held by Users

Calibration on Maker Token

Table 1: Parameters

Calibrated Parameters	Value	Interpretation	Moment
σ	0.62	Volatility of log-productivity	Volatility of token price
M	$5.4e-4$	Token mass	Price level of token
λ	1.06	Rate of arrival of users	Relation price-adoption
$\tilde{\mu}$	0.018	Mean of log-productivity	Relation price-adoption
α	2.48	Pareto shape parameter	Relation price-adoption
Normalized Parameters			
r	0.05	Risk-free interest rate	
η	0.50	Curvature utility function	
$\underline{\chi}$	1	Lower bound of users' ability	

Illustration



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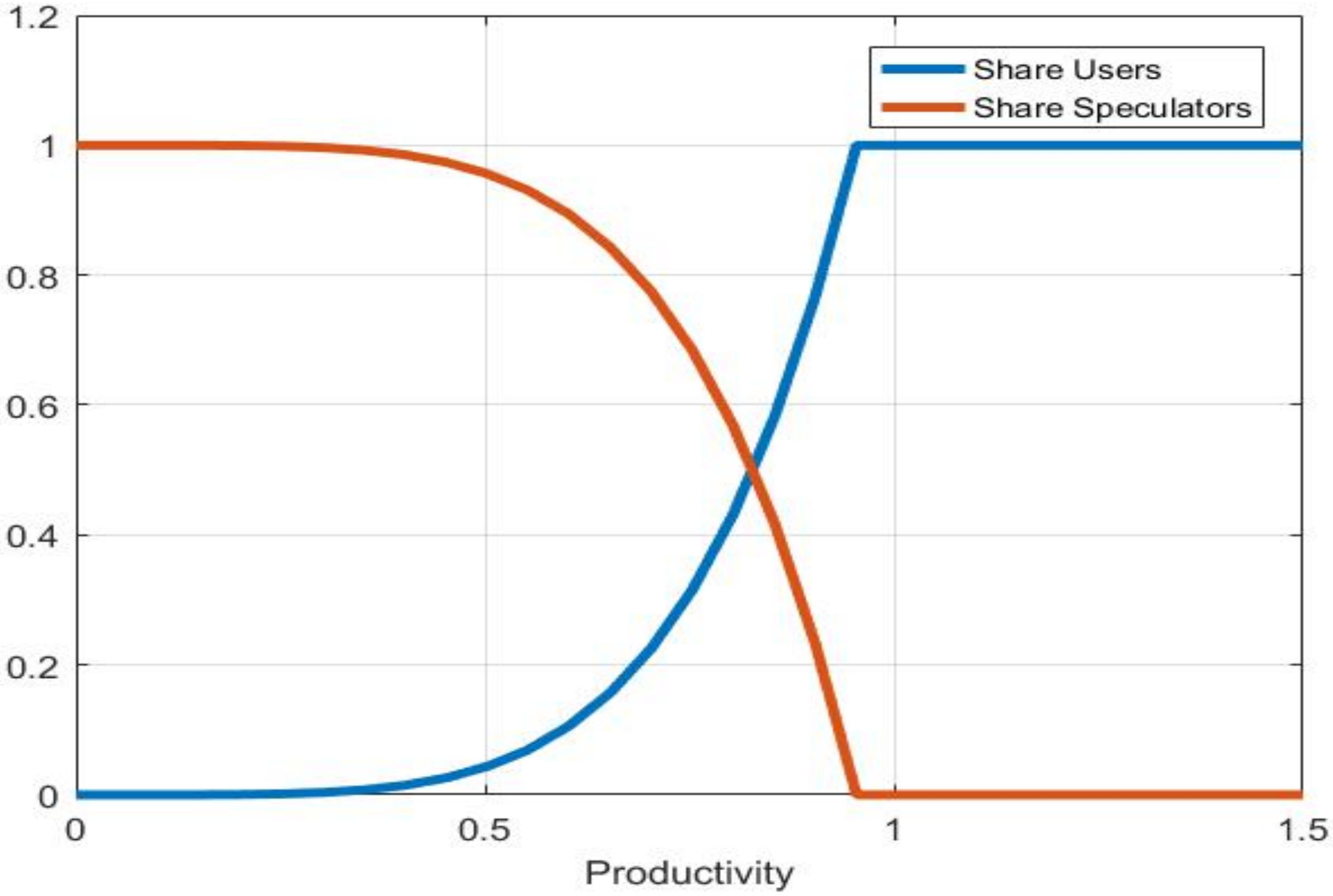
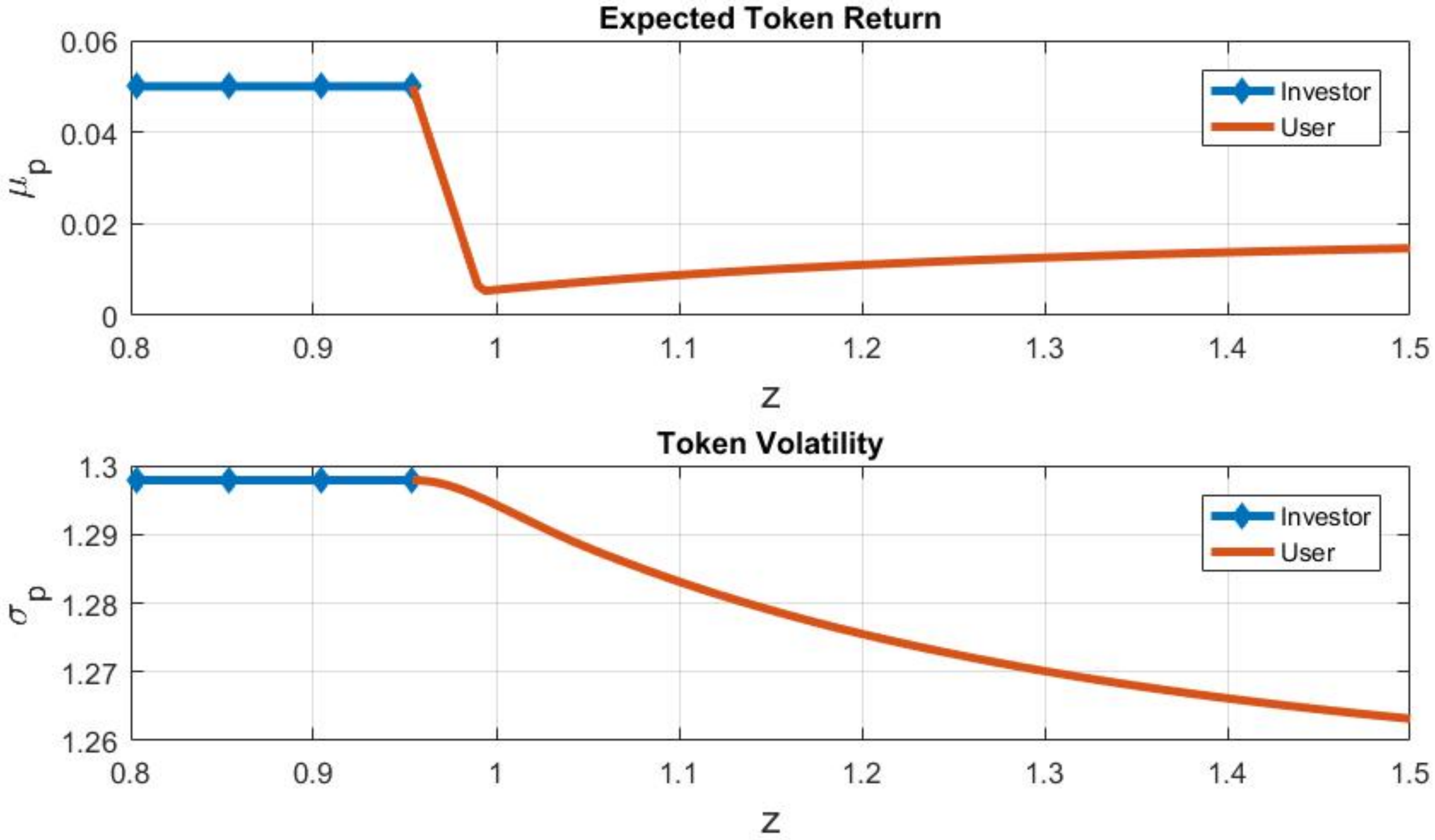


Returns


Expected Token Return
0.06
0.04
0.02
0
μ_p
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
z
Investor
User
Token Volatility
1.3
1.29
1.28
1.27
1.26
σ_p
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
z
Investor
User

Excess Volatility

- Price function in the investor regime

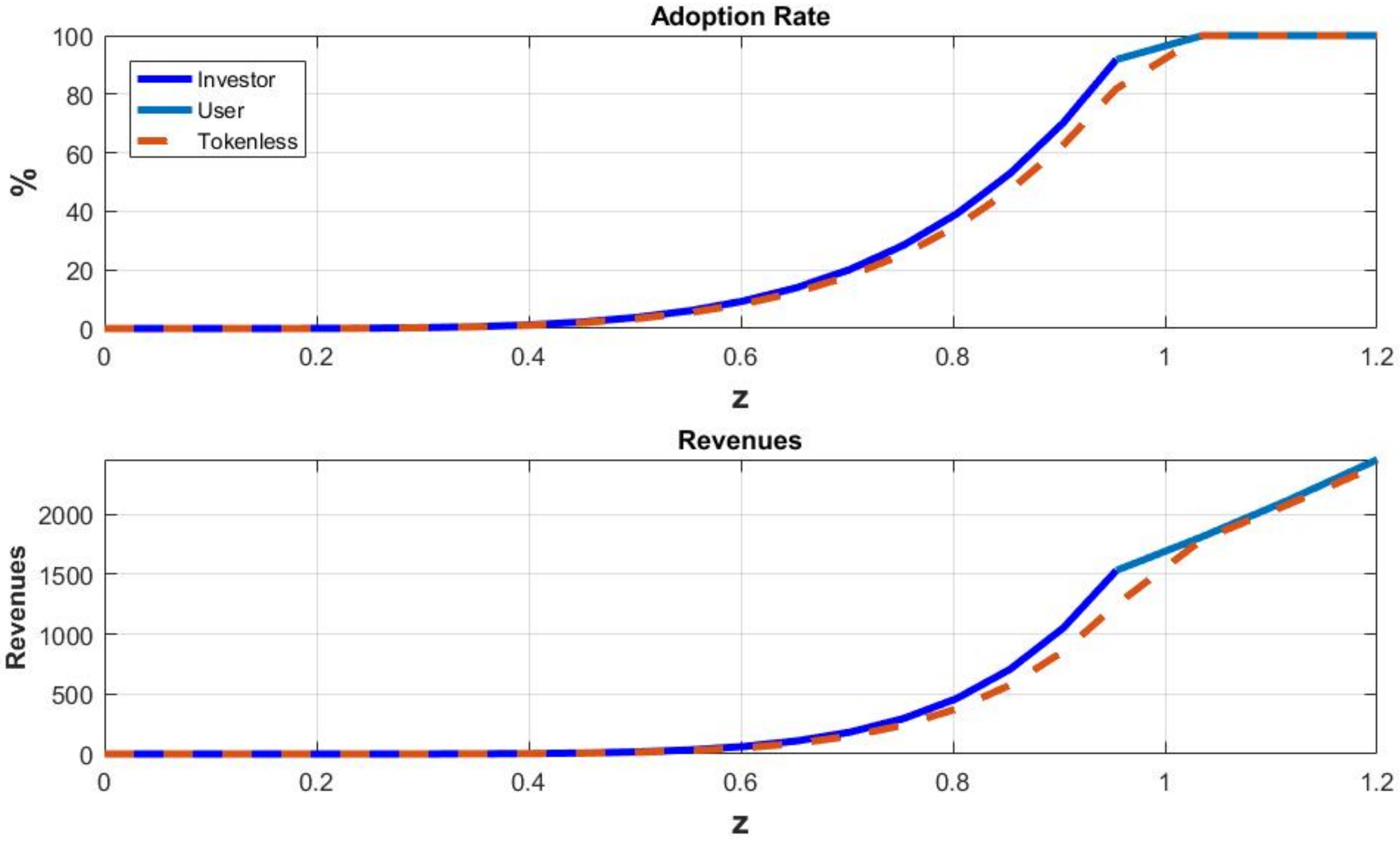
$$p(z) = \overline{p}(\underline{z}) \left(\frac{z}{\underline{z}} \right)^{\beta}, \text{ for all } z \leq \underline{z} = min\{z : p(z) \leq \overline{p}(z)\}$$

where $\beta > 1$.

- Convexity raises expected return above growth rate of productivity

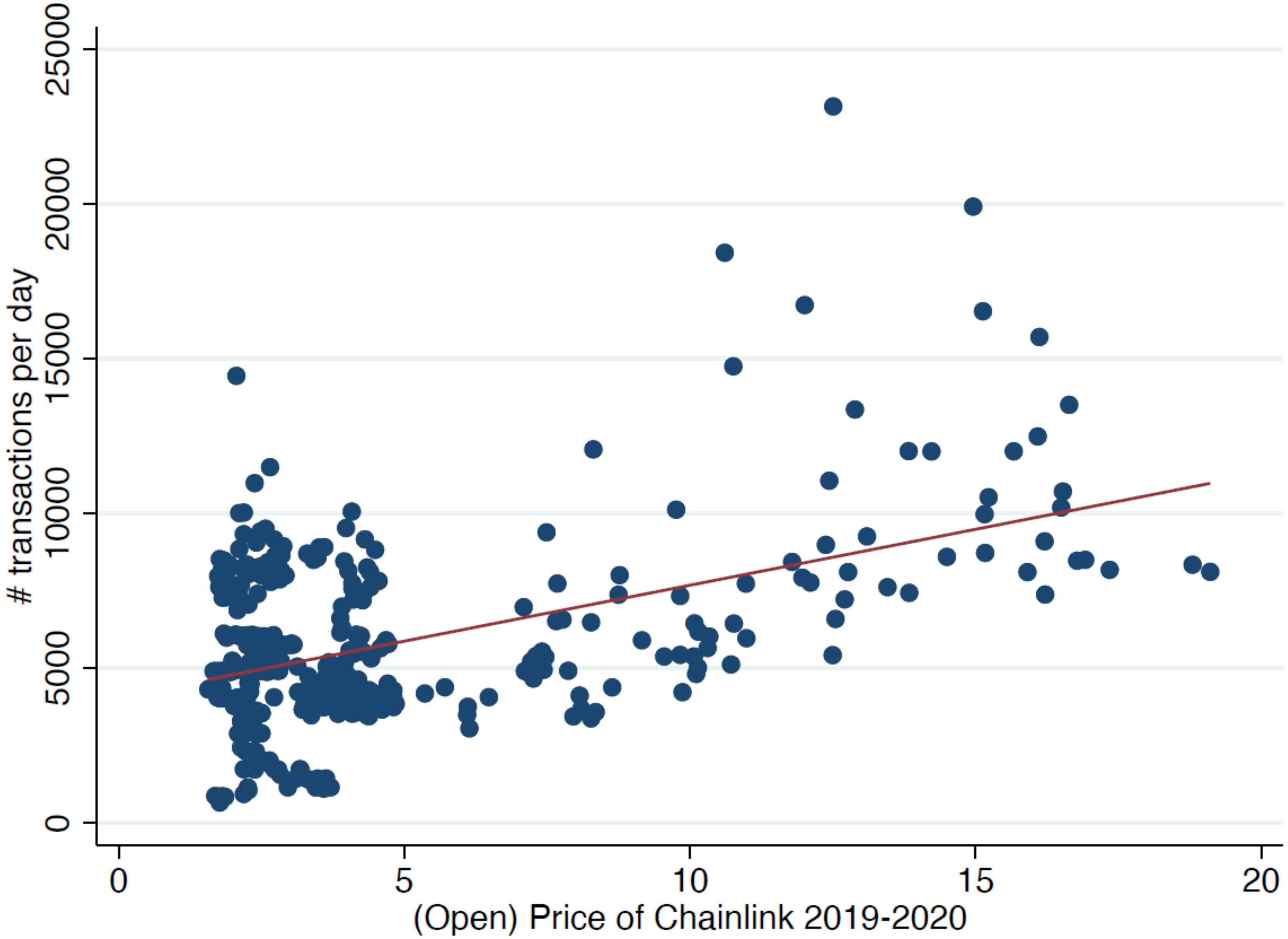
- Excess volatility provides investor with the required rate of return r

Token vs. Fiat



- Tokenization incentivizes adoption because it generates financial returns for users

Usage data for Chainlink



Token in Advance vs. in the Utility Function

- Cong et al. (2019) study model where tokens provide users with transactional benefits so that

$$v\left(p_t, \dot{p}_t, z_t\right) = \max_m \left\{ z_t u\left(p_t m\right) + \dot{p}_t m - r p_t m \right\}$$

- No investor regime because users' demand **always clears** the market

Conclusion

- Valuation framework that is based on fundamentals
- Microfound metrics used by investors such as the velocity of circulation of tokens
- Rationalize extreme volatility and high valuation of tokens early on during the adoption phase

Directions for further research

- Model scratches the surface:
  - ❑ Network effect
  - ❑ Optimal monetary policy
  - ❑ More complex designs for Tokens
  - ❑ Microfoundation with assymmetric information
  - ❑ Intermediation and Market Makers