Mathematical modelling and analysis of Impermanent Loss and Fees in Uniswap v3

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Summary of this presentation

✓ Working assumptions on the $X - Y$ exchange rate:

H$_0$**:** The prices within and outside the pool coincide at any time

H$_{Ito}$**:** The pool price process $(p_t)_t$ follows an Itô dynamics

$$\frac{\mathrm{d}p_t}{p_t} = \mu_t \mathrm{d}t + \sigma_t \mathrm{d}W_t$$

with possibly stochastic drift and volatility.

✓ Contributions:

- ✓ Under **H**$_0$, we provide a simple formula for the Y-value $V_P(t)$ of the pool position of a Liquidity Provider that has deposited some tokens X and Y on a set of arbitrary ranges.
- ✓ Under **H**$_0$, we show how any concave payoff can be explicitly replicated by depositing liquidity curve
- ✓ Under **H**$_{Ito}$, we provide an asymptotics formula for computing/predicting fees, as an integral of local times of the pool price process

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Related litterature

- ✓ Optimal execution and deep neural networks: [Jaimungal et al., 2023]
- ✓ Optimal liquidity provision: [Cartea et al., 2023a], [Fan et al., 2023], [Cartea et al., 2023b], [Fan et al., 2022]
- ✓ Optimal trade: [Cartea et al., 2022]
- ✓ Empirical study of Uniswap v3 pools: [Loesch et al., 2021]
- ✓ Impermanent Loss in Uniswap v3: [Lambert, 2021], [Boueri, 2022]
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Uniswap v2 - Principles

✓ Based on the Constant Function Market Makers:

$$\mathcal{I}(x, y) = x \cdot y.$$

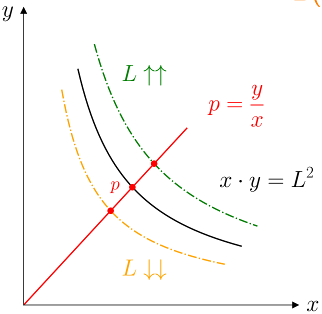


Figure: How prices and quantities evolve in a Uniswap v2 protocol

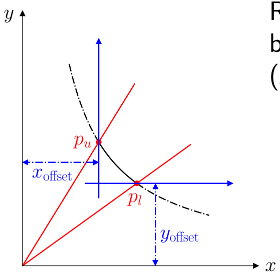
✓ Price/tokens/liquidity relations:

$$x = \frac{L}{\sqrt{p}} = \frac{L}{\pi} \quad \text{and} \quad y = L\sqrt{p} = L \cdot \pi.$$

where p is the pool price and π is the square root price

Uniswap v3 - Principles

- ✓ An LP provides an amount of liquidity L on a price range $[p_\ell, p_u)$: she deposits respective quantities x_r and y_r of tokens X and Y to the pool
- ✓ Tokens are used by swap operations as long as the price is in the range $[p_\ell, p_u)$. When the price reaches p_ℓ (resp. p_u), all reserves of tokens Y (resp. X) will have been depleted.
- ✓ On the price range $[p_\ell, p_u)$, the constant product rule $x \cdot y = L^2$ applies with the main difference that x and y denote *virtual* token reserves instead of real reserves.



Representation of the *virtual* numbers (x, y) and the *offset* numbers $(x_{\text{offset}}, y_{\text{offset}})$ of tokens:

$$\begin{cases} x = x_r + x_{\text{offset}}, \\ y = y_r + y_{\text{offset}}. \end{cases}$$

$$\Longrightarrow \left(x_r + \frac{L}{\pi_u}\right) \cdot (y_r + L \cdot \pi_\ell) = L^2.$$

Example of Liquidity curve

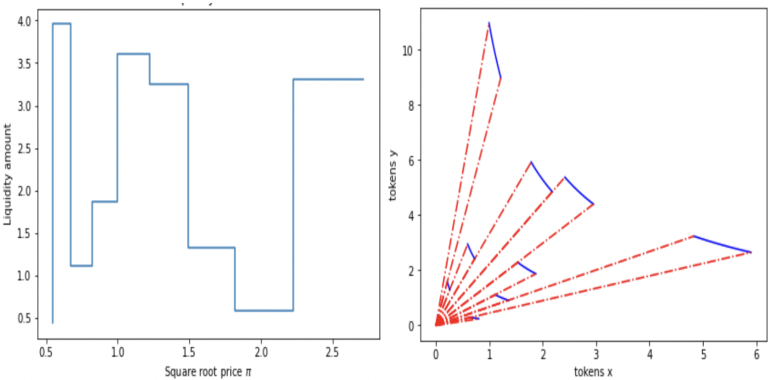


Figure: On the left: the liquidity curve.
On the right: the CPMM relation on each range.

Details about liquidity range and fees

- ✓ For liquidity events, the ranges of prices are restricted to *ticks*:

$$p(\tau) := \beta_{\mathrm{p}}^{\tau} \quad \text{where} \quad \beta_{\mathrm{p}} = 1.0001.$$

- ✓ An LP specify a tick range on which liquidity is to be added.
- ✓ Not all tick ranges can actually be used to update liquidity: the ranges are a multiple of a fixed number of ticks δ_π, which is ruled at the setting of the pool, according to the swap fees ϕ.

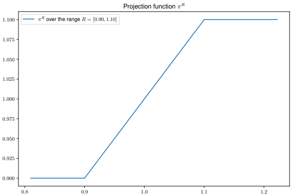
$$\boxed{\delta_\pi = 2, 10, 60, 200, \text{ according to swap fees } \phi = 0.01\%, 0.05\%, 0.3\%, 1\%}$$

- ✓ When a range is defined by two consecutive ticks $i \cdot \delta_\pi$ and $(i+1) \cdot \delta_\pi$, we refer to a *unitary range*.

Mathematical formulation of Mint/Burn by LP

Consider an *unitary square root price range* $[\pi_\ell, \pi_u)$ and define the projected square root price function:

$$\pi_0^R = \begin{cases} \pi_u \text{ if } \pi_u < \pi_0, \\ \pi_\ell \text{ if } \pi_0 < \pi_\ell, \\ \pi_0 \text{ otherwise.} \end{cases}$$



Lemma

The mint/burn operations (keeping the current square root price π_0 unchanged) on that range $R = [\pi_\ell, \pi_u)$ are described by a single formula that cover all situations

$$\Delta x_r = \Delta L \cdot \left(\frac{1}{\pi_0^R} - \frac{1}{\pi_u}\right) \quad \text{and} \quad \Delta y_r = \Delta L \cdot (\pi_0^R - \pi_\ell).$$

IL Computation

- ✓ Focus on a unitary price range $R = [\pi_\ell, \pi_u)$
- ✓ A LP adds an amount ΔL of liquidity on R, when the price in the pool is p_0
- ✓ Compare two strategies:
  - ✓ HODL strategy: $V_H =$ the value of a portfolio where tokens are held in a separate wallet (outside the pool)
  - ✓ Liquidity providing strategy: $V_P =$ the value of a portfolio where tokens are invested in a liquidity Uniswap v3 pool.
- ✓ Unique formula for $V_P - V_H$ covering all cases.

Theorem

Under $\mathbf{H}_0$, at time 1, taking Y as reference numéraire,

$$V_P - V_H = -\Delta L \cdot \left| \left(\pi_0^{[\pi_\ell,\pi_u)} - \pi_1^{[\pi_\ell,\pi_u)}\right) \cdot \left(1 - \frac{\pi_1^2}{\pi_0^{[\pi_\ell,\pi_u)} \cdot \pi_1^{[\pi_\ell,\pi_u)}}\right) \right| .$$

In $\mathbf{H}_0$ not satisfied, it is necessary to adjust the valuations of both strategies, by swapping all X tokens for Y tokens in the pool $\Rightarrow$ extra swap fees $\Rightarrow$ very limited impact on the results.

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Value of the LP position in the pool

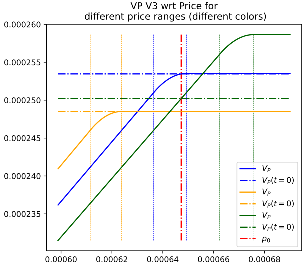
Theorem (V_P on a unitary range)

Assume $\mathbf{H}_0$. *Adding a liquidity* ΔL *on the unitary range* $R = [\pi_\ell, \pi_u)$ *at time* 0 *gives* Y*-value of the LP position equal to*

$$V_P(t) = \Delta L \cdot \left(\left(\frac{1}{\pi_t^R} - \frac{1}{\pi_u} \right) \cdot \pi_t^2 + (\pi_t^R - \pi_\ell) \right),$$

given as a function of the square root price π_t *at time* t.

Referred as *Covered call* on Guillaume Lambert's blog [Lambert, 2021].



Value of the pool for 3 different price ranges (orange, blue, green). The dashed vertical lines in each color represent the considered unitary ranges in the variable price p. The dashed horizontal lines represent the initial value of the pool for the price p_0 depicted in red.

Arbitrary liquidity curve

- ✓ Objective: better understand the link of the liquidity curve and the exposure to market changes

Theorem (V_P with an arbitrary liquidity curve)

Assume $\mathbf{H}_0$. Consider a liquidity provider adding a liquidity curve $(\Delta L_\pi)_\pi$ to the pool at time $t = 0$.
Then, its Y-value net of swap fees at time t is

$$V_P(t) = p_t \cdot \int_{\pi_t}^{+\infty} \frac{\Delta L_\pi}{\pi^2} d\pi + \int_0^{\pi_t} \Delta L_\pi d\pi$$

when the pool price is p_t.

- ✓ Remarkably simple!
- ✓ Valid for any occurence of swap trades or other mint/burn events.

Corollary (Greeks of V_P)

The Delta and Gamma of the Y-value of the LP position is

$$\Delta_P(t) \overset{\text{def}}{=} \frac{\partial V_P(t)}{\partial p_t} = \int_{\pi_t}^{+\infty} \frac{\Delta L_\pi}{\pi^2} d\pi,$$

$$\Gamma_P(t=0) \overset{\text{def}}{=} \frac{\partial^2 V_P(t)}{\partial p_t^2} = -\frac{\Delta L_{\pi_t}}{2\pi_t^3}.$$

In particular, such a position is always concave in the spot rate (Gamma negative).

✓ Allows for risk management of crypto portfolios (greeks, delta and gamma hedging. . .)

Implicit liquidity curve

Theorem (Liquidity curve generating a given payoff)

Assume $\mathbf{H}_0$*. Consider a concave payoff* $h : \mathbb{R}^+ \mapsto \mathbb{R}$ *which we assume to be* C^3 *and linear for small and large values. Then, consider a strategy depositing the liquidity curve*

$$\Delta L_R := \left(-h''\left(\pi_\ell \cdot \pi_u\right)\right) \cdot \left(\pi_u + \pi_\ell\right) \cdot \pi_\ell \cdot \pi_u$$

at time 0 on each range $R = [\pi_\ell, \pi_u)$*. In addition, add to the position quantities* x_0 *of tokens* X *and* y_0 *of tokens* Y *outside the pool, with*

$$x_0 = h'(p_0) - \sum_{R=[\pi_\ell,\pi_u)} \Delta L_R \cdot \left(\frac{1}{\pi_0^R} - \frac{1}{\pi_u}\right),$$

$$y_0 = h(p_0) - h'(p_0) \cdot p_0 + \sum_{R=[\pi_\ell,\pi_u)} \Delta L_R \cdot \left(\pi_0^R - \pi_\ell\right).$$

Then

$$\left|h(p_T) - \left(V_P(T) + x_0 \cdot p_T + y_0\right)\right| \leq C \cdot \delta_\pi \cdot (\beta_{\mathrm{p}} - 1), \quad a.s.$$

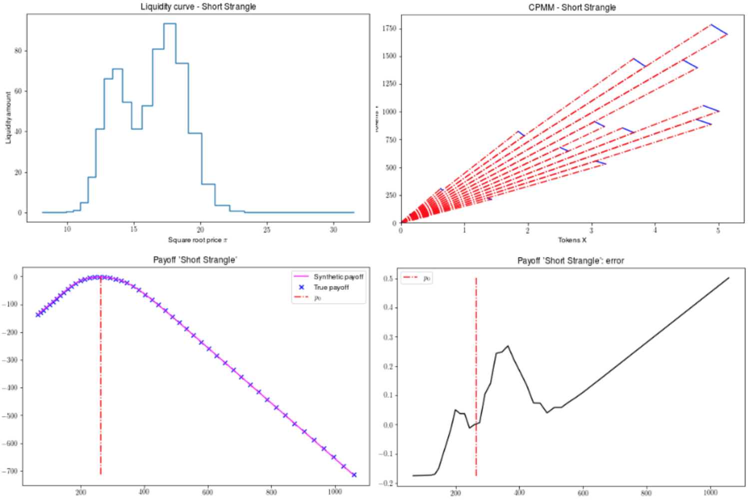


Figure: Illustration when h corresponds to a short strangle (minus a put with strike $K = p_0/1.3$ and minus a call with strike $K = 1.3 \cdot p_0$. Both options are considered with a maturity $\tau = 0.1$ and a Black-Scholes volatility equal to 50%). Top left: the liquidity curve ΔL_R. Top right: the CPMM representation. Bottom left: the payoff and its replication. Bottom right: the reconstruction error.

Sketch of proof

Carr-Madan formula:

$$h(p_T) = h(p_0) + h'(p_0)\cdot(p_T - p_0) + \int_{p_0}^{+\infty} h''(K)(p_T - K)_+ \mathrm{d}K + \int_0^{p_0} h''(K)(K - p_T)_+ \mathrm{d}K.$$

Then

$$h(p_T) = h(p_0) + h'(p_0)\cdot(p_T - p_0) + \sum_{R=[\pi_\ell,\pi_u)\subset[\pi_0,\infty)} h''(\pi_\ell\cdot\pi_u)\cdot\left(\pi_T^2 - \pi_\ell\cdot\pi_u\right)_+\cdot\left(\pi_u^2 - \pi_\ell^2\right)$$
$$+ \sum_{R=[\pi_\ell,\pi_u)\subset[0,\pi_0)} h''(\pi_\ell\cdot\pi_u)\cdot\left(\pi_\ell\cdot\pi_u - \pi_T^2\right)_+\cdot\left(\pi_u^2 - \pi_\ell^2\right) + O\left(\delta_\pi\cdot(\beta_\mathrm{p} - 1)\right)$$

$$= h(p_0) + h'(p_0)\cdot(p_T - p_0)$$
$$+ \sum_{R=[\pi_\ell,\pi_u)} \left(-h''(\pi_\ell\cdot\pi_u)\right)\cdot(\pi_u + \pi_\ell)\cdot\pi_\ell\cdot\pi_u\cdot\left(V_P^R(\pi_T) - V_H^R(\pi_T)\right) + \dots$$

Swap traders and fees bookkeeping

- ✓ Contrarily to Uniswap v2 pools, fees are not considered as additional tokens in the reserves of the Uniswap v3 pool.
- ✓ Fees are concentrated specifically on each range
- ✓ Given a range $R = [\pi_\ell, \pi_u)$, the *fees* in the pool are tracked by two accumulators Φ_R^X and Φ_R^Y that are updated at every transaction (`feeGrowthGlobal0X128` and `feeGrowthGlobal1X128` in the source code).
- ✓ Give amounts of fees per unit of liquidity.
- ✓ These accumulators are recovered by

$$\Phi_R^X = \Phi^X - \varphi_{\mathrm{b}}^X(\pi_\ell) - \varphi_{\mathrm{a}}^X(\pi_u) \quad \text{and} \quad \Phi_R^Y = \Phi^Y - \varphi_{\mathrm{b}}^Y(\pi_\ell) - \varphi_{\mathrm{a}}^Y(\pi_u),$$

which are implemented in the `getFeeGrowthInside` method of TICK.SOL and invoked when the position is updated (such as in the `_updatePosition` method of UNISWAPV3POOL.SOL).

Analytical formula for fees

Theorem (Fees in X and Y accumulated over $[0, T]$)

Consider a LP depositing a liquidity curve $(\Delta L_\pi)_\pi$ at time 0. Assume

- ✓ *$(\Delta L_\pi)_\pi$ has a finite support*
- ✓ *$\mathbf{H}_{Ito}$ and some mild assumptions on μ_t, σ_t*
- ✓ *the swap trades cause the price process $(p_t)_t$ to move from one tick to another*

Then

$$\mathrm{Fees}^X_{0\to T} = \sum_{R=[\pi_\ell,\pi_u)} \Delta L_\pi \cdot \left(\Phi^X_R(T) - \Phi^X_R(0)\right),$$

$$\mathrm{Fees}^Y_{0\to T} = \sum_{R=[\pi_\ell,\pi_u)} \Delta L_\pi \cdot \left(\Phi^Y_R(T) - \Phi^Y_R(0)\right),$$

$$\lim_{\beta_\mathrm{p}\downarrow 1} (\beta_\mathrm{p} - 1) \cdot \mathrm{Fees}^X_{0\to T} \stackrel{\mathbb{P}}{=} \frac{\phi}{1-\phi} \cdot \int_0^{+\infty} \Delta L_{b^{\frac{1}{2}}} \frac{A^b_T(p)}{4 \cdot b^{5/2}} \mathrm{d}b,$$

$$\lim_{\beta_\mathrm{p}\downarrow 1} (\beta_\mathrm{p} - 1) \cdot \mathrm{Fees}^Y_{0\to T} \stackrel{\mathbb{P}}{=} \frac{\phi}{1-\phi} \cdot \int_0^{+\infty} \Delta L_{b^{\frac{1}{2}}} \frac{A^b_T(p)}{4 \cdot b^{3/2}} \mathrm{d}b.$$

where $A^b_T(p)$ is the local time of p at level b and time T.

Sketch of proof

The amount of fees in tokens X and Y accumulated over the period per unit of liquidity on the range $R = [\pi_\ell, \pi_u)$ is

$$\begin{aligned}
\Delta\Phi_R^X &= \sum_{\text{swap } X \text{ for } Y \text{ at } t_i \text{ in } R} \frac{a_i^X}{1-\phi}\cdot\frac{\phi}{L_i} \\
&= \sum_{\text{swap } X \text{ for } Y \text{ at } t_i \text{ in } R} \frac{\frac{L_i}{\pi_{t_i}\beta_{\mathrm{p}}^{-1/2}} - \frac{L}{\pi_{t_i}}}{1-\phi}\cdot\frac{\phi}{L_i} \\
&= \frac{(\beta_{\mathrm{p}}^{1/2}-1)\cdot\phi}{2\cdot(1-\phi)}\cdot\sum_{\text{swap at time } t_i\in[0,T]} \mathbf{1}_{\pi_{t_i}\in R}\cdot\frac{1}{\pi_{t_i}} + \cdots \\
&= \frac{(\beta_{\mathrm{p}}^{1/2}-1)\cdot\phi}{2\cdot(1-\phi)}\cdot\sum_{\text{swap at time } t_i\in[0,T]} \mathbf{1}_{\pi_{t_i}\in R}\cdot\frac{1}{\pi_{t_i}}\cdot\frac{\int_{t_i}^{t_{i+1}}\sigma_t^2\mathrm{d}t}{(\log(\beta_{\mathrm{p}}))^2} + \cdots \\
&= \frac{\phi}{4\cdot(1-\phi)\cdot(\beta_{\mathrm{p}}-1)}\cdot\int_0^T \mathbf{1}_{\pi_t\in R}\frac{1}{\pi_t}\cdot\sigma_t^2\mathrm{d}t + \cdots \\
&= \frac{\phi}{(1-\phi)\cdot(\beta_{\mathrm{p}}-1)}\cdot\int_R \frac{A_T^a(\pi)}{a^3}\mathrm{d}a + \cdots
\end{aligned}$$

Confirmation from experiments

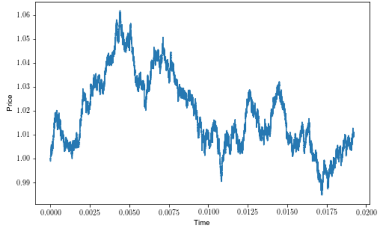


Figure: Sample path of GBM for p, with $p_0 = 1$, $\sigma = 40\%$ and drift $\mu = 5\%$

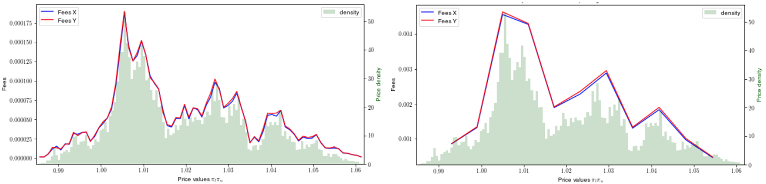


Figure: Left axis: the exact fees collected in tokens X and Y. Right axis: occupation density of p. Left: when $\delta_\pi = 10$. Right: when $\delta_\pi = 60$.

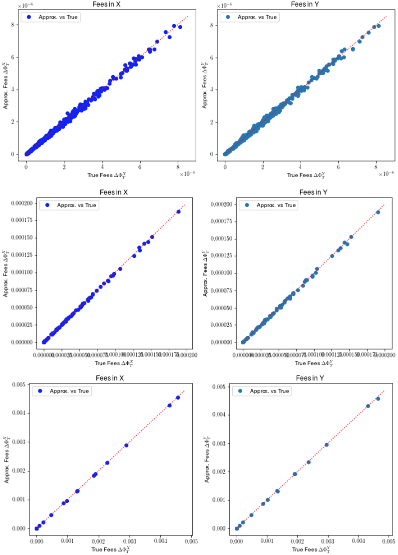


Figure: True collected fees versus their approximations. From top to bottom: $\delta_\pi = 2, 10, 60$. Each point corresponds to the amount of swap fee on a range; smaller values of δ_π have more ranges hence more points.

Theorem

Consider the Y-value of the approximated collected swap fees:

$$\mathfrak{F}\big((\Delta L_\pi)_\pi, (\sigma_t)_t\big) \overset{\text{def}}{=} \frac{1}{(\beta_{\mathrm{p}}-1)}\mathbb{E}^\star\Bigg[\lim_{\beta_{\mathrm{p}}\downarrow 1}(\beta_{\mathrm{p}}-1)\cdot \mathrm{Fees}^X_{0\to T}\cdot p_T + \lim_{\beta_{\mathrm{p}}\downarrow 1}(\beta_{\mathrm{p}}-1)\cdot \mathrm{Fees}^Y_{0\to T}\Bigg].$$

Assume the existence of a risk-neutral valuation rule under $\mathbb{P}^\star$ with unit discounted factor. We have

$$\begin{aligned}&(\beta_{\mathrm{p}}-1)\cdot\mathfrak{F}\big((\Delta L_\pi)_\pi, (\sigma_t)_t\big)\\ &=\frac{\phi}{(1-\phi)}\cdot\int_0^{+\infty}\Delta L_{b^{\frac12}}\frac{\mathbb{E}^\star\left[A_T^b(p)\right]}{2\cdot b^{3/2}}\mathrm{d}b\\ &=\frac{\phi}{(1-\phi)}\cdot\left(\int_0^{p_0}\Delta L_{b^{\frac12}}\cdot\frac{\mathrm{Put}_{t=0}(T,b)}{b^{3/2}}\mathrm{d}b+\int_{p_0}^{+\infty}\Delta L_{b^{\frac12}}\cdot\frac{\mathrm{Call}_{t=0}(T,b)}{b^{3/2}}\mathrm{d}b\right).\end{aligned}$$

✓ Similar to a Carr-Madan formula

✓ Allows for comparing fees with CEX options and studying CEX-DEX arbitrage

Computation in Black-Scholes model

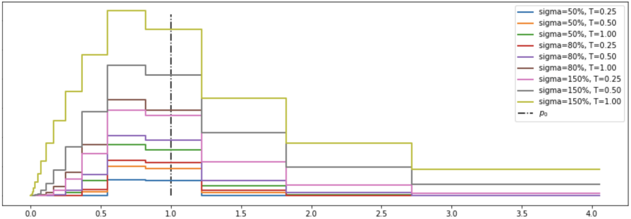
Theorem

In the Black-Scholes model

$$\mathbb{E}^\star\left[\lim_{\beta_{\rm p}\downarrow 1}(\beta_{\rm p}-1)\cdot\mathrm{Fees}^X_{0\to T}\cdot p_T+\lim_{\beta_{\rm p}\downarrow 1}(\beta_{\rm p}-1)\cdot\mathrm{Fees}^Y_{0\to T}\right]$$

$$=\frac{\phi\cdot\sigma^2}{2\cdot(1-\phi)}\cdot\int_0^T \pi_0 e^{-\frac{\sigma^2}{8}t}\cdot\left(\mathcal{N}\left(\frac{1}{\sigma\sqrt{t}}\cdot\ln\left(\frac{p_0}{p_l}\right)-\frac{1}{2}\cdot\sigma\sqrt{t}\right)\right.$$

$$\left.-\mathcal{N}\left(\frac{1}{\sigma\sqrt{t}}\cdot\ln\left(\frac{p_0}{p_u}\right)-\frac{1}{2}\cdot\sigma\sqrt{t}\right)\right)\mathrm{d}t.$$



Conclusion

Our contributions

- ✓ Analytical representations of the pool value (Uniswap v3), for any arbitrary liquidity curve
- ✓ Any concave payoff can be replicated by a v3 pool
- ✓ Proxy formula for the fees, in a quite general setting

Work in progress

- ✓ Statistical study of swap fees and comparison with the proxy formula
- ✓ Quantitative study of arbitrage opportunities between CEX-DEX, options, spots
- ✓ More to come out from Kaiko products

References I

Bichuch, M. and Feinstein, Z. (2023).
Implied volatility in decentralized finance from automated market makers.
Presentation at SIAM Conference on Financial Mathematics and Engineering, Philadelphia, 6-9 June 2023.

Boueri, N. (2022).
G3M impermanent loss dynamics.
arXiv preprint arXiv:2108.06593.

Cartea, Á., Drissi, F., and Monga, M. (2022).
Decentralised finance and automated market making: Execution and speculation.
Available at SSRN 4144743.

Cartea, Á., Drissi, F., and Monga, M. (2023a).
Decentralised finance and automated market making: Predictable loss and optimal liquidity provision.
arXiv preprint arXiv:2309.08431.

References II

Cartea, Á., Drissi, F., Sánchez-Betancourt, L., Siska, D., and Szpruch, L. (2023b).
Automated market makers designs beyond constant functions.
Available at SSRN 4459177.

Fan, Z., Marmolejo-Cossío, F., Altschuler, B., Sun, H., Wang, X., and Parkes, D. C. (2022).
Differential liquidity provision in Uniswap v3 and implications for contract design.

Fan, Z., Marmolejo-Cossío, F., Moroz, D. J., Neuder, M., Rao, R., and Parkes, D. C. (2023).
Strategic liquidity provision in Uniswap v3.
arXiv preprint arXiv:2106.12033.

Jaimungal, S., Saporito, Y. F., Souza, M. O., and Thamsten, Y. (2023).
Optimal trading in automatic market makers with deep learning.
arXiv preprint arXiv:2304.02180.

References III

Lambert, G. (2021).
Uniswap v3 LP tokens as perpetual put and call options.
Medium.
https://medium.com/@lambert-guillaume/uniswap-v3-lp-tokens-as-perpetual-put-and-call-options

Loesch, S., Hindman, N., Richardson, M. B., and Welch, N. (2021).
Impermanent loss in Uniswap v3.
arXiv preprint arXiv:2111.09192.