Decentralised Finance and Automated Market Making: Execution and Speculation

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Blockchain@X-OMI Workshop

21 September 2023

UNIVERSITY OF OXFORD

OXFORD-MAN INSTITUTE

Introduction

Motivation

- New paradigm in the design of trading venues
- Poised to challenge electronic exchanges in all asset classes

Our contributions

- New class of Optimal Execution problems
- Data analysis and backtesting on real data

Constant Function Market Makers

In charge of an asset pair X and Y

Available liquidity q^X and q^Y

Deterministic trading function $f\left(q^X, q^Y\right)$

Liquidity Takers

- They send a quantity y of asset Y to the CPMM to receive a quantity x of X according to the trading function

$$q^X \, q^Y = \left(q^X - x\right) \left(q^Y + y\right) = \kappa^2 \,,$$

- They receive

$$x = \frac{\kappa^2}{q^Y} - \frac{\kappa^2}{q^Y + y} = \varphi\left(q^Y\right) - \varphi\left(q^Y + y\right)$$

- For each unit of Y that they sell they receive

$$\frac{x}{y} = \frac{\varphi\left(q^Y\right) - \varphi\left(q^Y + y\right)}{y} \xrightarrow{y \longrightarrow 0} -\varphi'\left(q^Y\right) = \frac{\kappa^2}{\left(q^Y\right)^2} = \frac{q^X}{q^Y} =: Z$$

Execution Costs

Execution price

$$\tilde{Z}(y) = \frac{\varphi\left(q^Y\right) - \varphi\left(q^Y + y\right)}{y}$$

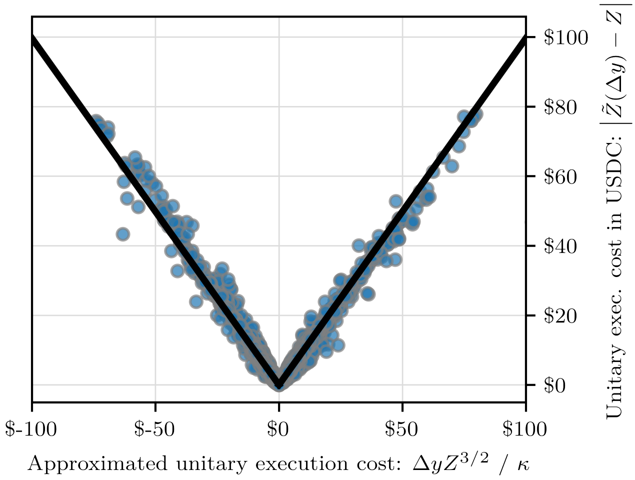
Instantaneous rate

$$Z = -\varphi'\left(q^Y\right)$$

Execution costs

$$\left|\tilde{Z}(y) - Z\right| \approx \frac{1}{\kappa} Z^{3/2} \left|y\right|$$

Execution Costs



Liquidity Providers

- **Liquidity provision condition:** liquidity providers cannot change the instantaneous rate Z

  Key difference from LOB markets

- They change the pool size by providing liquidity in both assets and the value of κ changes

$$\frac{q^X + x}{q^Y + y} = \frac{q^X}{q^Y}$$

Price Dynamics

We consider two trading venues

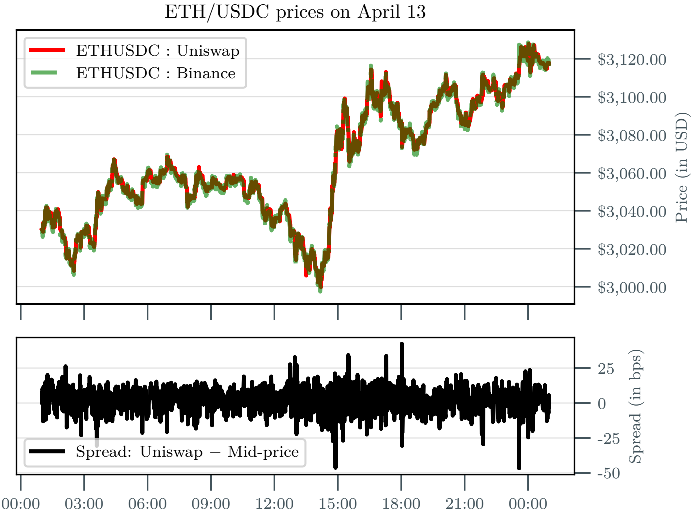
- Market reference price

$$dS_t = \sigma\, S_t\, dW_t\ ,$$

- Price implied by the CPMM

$$dZ_t = \beta\,(S_t - Z_t)\,dt + \gamma\, Z_t\, dB_t$$

Price Dynamics



Extension to multiple pairs

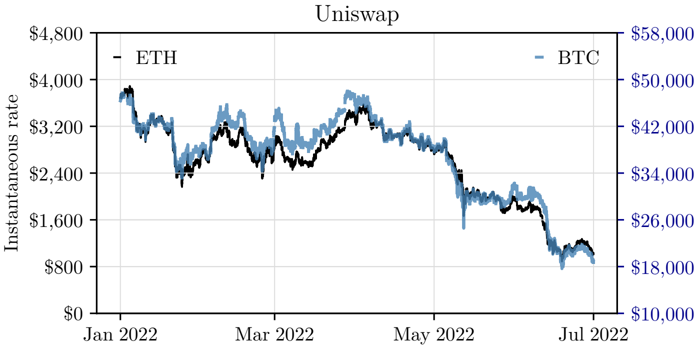


Figure: Exchange rates USDC/ETH and USDC/BTC in Uniswap v3 between January 2022 and June 2022.

Optimal Execution Problem

- Agent's holdings of asset Y:

$$d\tilde{y}_t = -\nu_t \, dt \,,$$

- Agent's holdings of aset X:

$$d\tilde{x}_t = \left(Z_t - \frac{\eta}{\kappa} \, Z_t^{3/2} \, \nu_t \right) \, \nu_t \, dt \,,$$

Constant κ Assumption

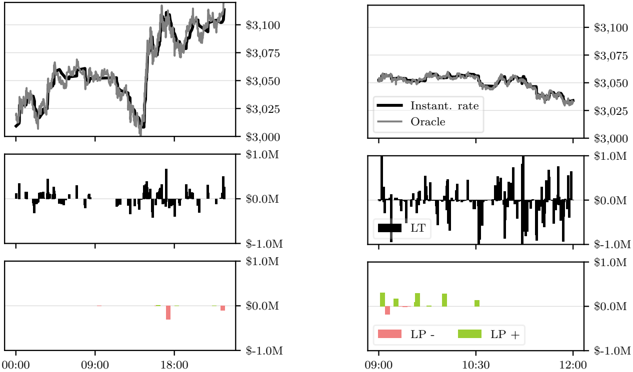
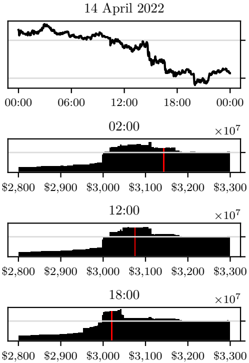
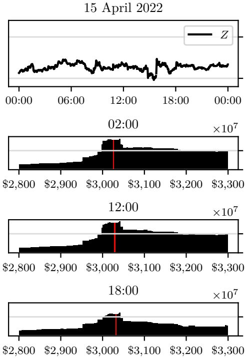


Figure: Rate and LP dynamics in AMMs.

Uniswap v3 Compatibility



(a) **Top**: ETH/USDC rates on 14 April 2022 **Other plots**: Pool depth κ at different times. The red bar is the instantaneous rate.



(b) **Top**: ETH/USDC rates on 15 April 2022 **Other plots**: Pool depth κ at different times. The red bar is the instantaneous rate.

Optimization Problem

- Performance criteria of strategy ν:

$$u^\nu(t,\tilde{x},\tilde{y},Z,S) = \mathbb{E}_{t,\tilde{x},\tilde{y},Z,S}\left[\tilde{x}_T^\nu + \tilde{y}_T^\nu\, Z_T - \alpha\,(\tilde{y}_T^\nu)^2 - \phi\int_t^T (\tilde{y}_s^\nu)^2\, ds\right]\,,$$

- Value function:

$$u(t,\tilde{x},\tilde{y},Z,S) = \sup_{\nu\in\mathcal{A}}\{u^\nu(t,\tilde{x},\tilde{y},Z,S)\}\,.$$

Closed-form Approximation Strategy

We use stochastic control tools and approximation techniques to obtain the closed-form approximation strategy

$$\nu^\star = \underbrace{-\frac{\kappa}{\eta}\, Z^{-3/2}\, A(t,Z)\, \tilde{y}}_{\text{Execution}} + \underbrace{\frac{\kappa}{2\,\eta}\, Z^{-3/2}\, B(t,Z)\,(S-Z)}_{\text{Speculation}},$$

- A is a positive function and helps mitigating inventory risk arising from $\tilde{y}$
- B is a positive function and exploits difference between the reference rate S and Z

Strategy Specifics

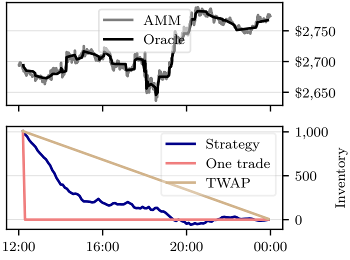
- In-sample parameters calibration
- Out-of-sample testing
- Initial inventory set to 50% of the observed hourly volume
- Agent's frequency is the observed average trading frequency over the in-sample period
- Testing execution and speculation strategy

Calibration of Parameters

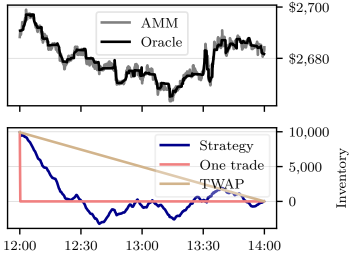
	ETH/USDC	ETH/DAI
$\hat{\sigma}$	0.045 $\mathrm{day}^{-1/2}$	0.053 $\mathrm{day}^{-1/2}$
$\hat{\gamma}$	0.034 $\mathrm{day}^{-1/2}$	0.027 $\mathrm{day}^{-1/2}$
$\hat{\beta}$	657.9 day^{-1}	14.78 day^{-1}

Table: Parameter estimates for dynamics of Z and S with data between noon 15 March 2022 and noon 16 March 2022.

Performance: Execution and Speculation



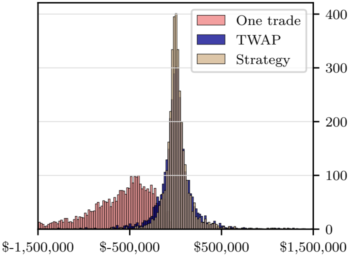
(c) ETH/DAI



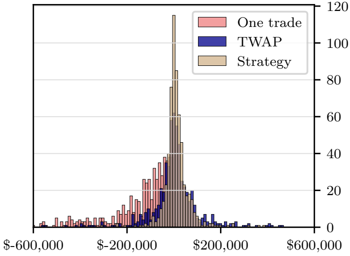
(d) ETH/USDC

Figure: Liquidation strategies starting at noon on 16 March 2022.

Performance: Execution and Speculation



(a) Final PnL distribution of strategies for ETH/USDC (3,635 executions).



(b) Final PnL distribution of strategies for ETH/DAI (607 executions).

Figure: PnL distribution.

Performance: Execution and Speculation

	Avg. PnL	Std. dev.	Avg. num. trades	Avg. fees
Single order	$-956{,}298$	1,963,014	1	2,538
TWAP	3,998	217,001	439	10,529
Liquidation	27,185	288,518	439	11,885

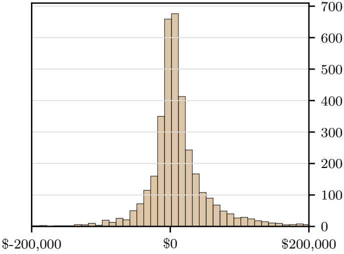
Table: Performance and fees for ETH/USDC (3,635 executions). The Average PnL does not include fees

Performance: Execution and Speculation

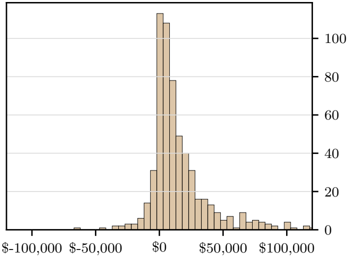
	Avg. PnL	Std. dev.	Avg. num. trades	Avg. fees
Single order	−233,390	428,688	1	634
TWAP	1,875	170,008	108	1,217
Liquidation	12,240	63,605	108	1,782

Table: Performance and fees for ETH/DAI (607 executions). The Average PnL does not include fees

Performance: Pure Speculative Strategy



(a) Statistical arbitrage PnL distribution for ETH/USDC.



(b) Statistical arbitrage PnL distribution for ETH/DAI.

Figure: Statistical arbitrage PnL distribution.

Performance: Pure Speculative Strategy

	Avg. PnL	Std. dev.	Avg. num. trades	Avg. fees
ETH/USDC	22,693	190,789	439	7,111
ETH/DAI	20,886	54,043	108	2,082

Table: Performance and fees for the **speculative strategy** for pairs ETH/USDC (3,635 executions) and ETH/DAI (607 executions). The Average PnL does not include fees

Conclusions

- We used Uniswap v3 data to analyse rate, liquidity, and execution costs of CPMMs
- We introduced a model where price are formed in an alternative trading venue
- We derived an approximated optimal trading strategy in closed-form
- We back-tested our trading strategy using real data

Thank you for listening!

Any questions?